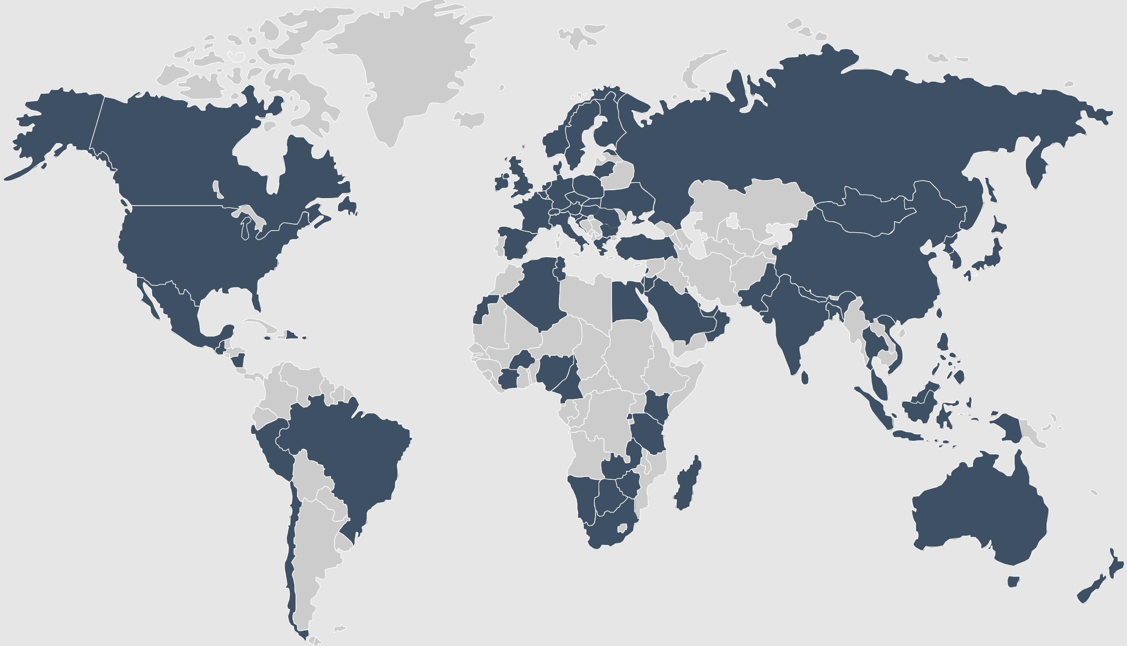

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Quantitative Methods

The Future Value of a Single Cash Flow

$$FV_N = PV\,(1+r)^N$$

The Present Value of a Single Cash Flow

$$PV = \frac{FV}{(1+r)^N}$$

$$PV_{\text{Annuity Due}} = PV_{\text{Ordinary Annuity}} \times (1+r)$$
$$FV_{\text{Annuity Due}} = FV_{\text{Ordinary Annuity}} \times (1+r)$$

Present Value of a Perpetuity

$$PV(\text{perpetuity}) = \frac{PMT}{I/Y}$$

Continuous Compounding and Future Values

$$FV_N = PVe^{r_s * N}$$

Effective Annual Rates

$$EAR = (1 + \text{Periodic interest rate})^N - 1$$

Net Present Value

$$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

where

CF_t = the expected net cash flow at time t
N = the investment's projected life
r = the discount rate or appropriate cost of capital

Bank Discount Yield

$$r_{BD} = \frac{D}{F} \times \frac{360}{t}$$

where:

r_{BD} = the annualized yield on a bank discount basis.
D = the dollar discount (face value – purchase price)
F = the face value of the bill
t = number of days remaining until maturity

Holding Period Yield

$$HPY = \frac{P_1 - P_0 + D_1}{P_0} = \frac{P_1 + D_1}{P_0} - 1$$

where:

P_0 = initial price of the investment.
P_1 = price received from the instrument at maturity/sale.
D_1 = interest or dividend received from the investment.

Effective Annual Yield

$EAY = (1 + HPY)^{365/t} - 1$

where:
HPY = holding period yield
t = numbers of days remaining till maturity

$HPY = (1 + EAY)^{t/365} - 1$

Money Market Yield

$$R_{MM} = \frac{360 \times r_{BD}}{360 - (t \times r_{BD})}$$

$R_{MM} = HPY \times (360/t)$

Bond Equalent Yield

BEY = [(1 + EAY) ^ 0.5 - 1]

Population Mean

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

Where,
X_i = is the *i*th observation.

Sample Mean

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Geometric Mean

$$1 + R_G = \sqrt[T]{(1 + R_1) \times (1 + R_2) \times \ldots \times (1 + R_T)} \quad \text{OR} \quad G = \sqrt[n]{X_1 X_2 X_3 \ldots X_n}$$

with $X_i \geq 0$ for $i = 1, 2, \ldots, n$.

$$R_G = \left[\prod_{t=1}^{T} (1 + R_t)\right]^{\frac{1}{T}} - 1$$

Harmonic Mean

Harmonic mean: $\overline{X}_H = \dfrac{N}{\sum_{i=1}^{N} \frac{1}{X_i}}$ with $X_i > 0$ for $i = 1, 2, \ldots, N$.

Percentiles

$$L_y = \frac{(n+1)\,y}{100}$$

where:
y = percentage point at which we are dividing the distribution
L_y = location (L) of the percentile (P_y) in the data set sorted in ascending order

Range

Range = Maximum value - Minimum value

Mean Absolute Deviation

$$MAD = \frac{\sum_{i=1}^{n} |X_i - \bar{X}|}{n}$$

Where:
n = number of items in the data set
$\bar{X}$ = the arithmetic mean of the sample

Population Variance

$$\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$$

where:
X_i = observation i
μ = population mean
N = size of the population

Population Standard Deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}}$$

Sample Variance

$$\text{Sample variance} = s^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$

where:
n = sample size.

Sample Standard Deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}}$$

Coefficient of Variation

$$\text{Coefficient of variation} = \frac{s}{\bar{X}}$$

where:
s = sample standard deviation
$\bar{X}$ = the sample mean.

Sharpe Ratio

$$\text{Sharpe ratio} = \frac{\bar{r}_p - r_f}{s_p}$$

where:
$\bar{r}_p$ = mean portfolio return
r_f = risk-free return
s_p = standard deviation of portfolio returns

Sample skewness, also known as sample relative skewness, is calculated as:

$$S_K = \left[\frac{n}{(n-1)(n-2)}\right]\frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{s^3}$$

As n becomes large, the expression reduces to the mean cubed deviation.

$$S_K \approx \frac{1}{n}\frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{s^3}$$

where:
s = sample standard deviation

Sample Kurtosis uses standard deviations to the fourth power. Sample excess kurtosis is calculated as:

$$K_E = \left(\frac{n(n+1)}{(n-1)(n-2)(n-3)} \frac{\sum_{i=1}^{n} (X_i - \bar{X})^4}{s^4} \right) - \frac{3(n-1)^2}{(n-2)(n-3)}$$

As n becomes large the equation simplifies to:

$$K_E \approx \frac{1}{n} \frac{\sum_{i=1}^{n} (X_i - \bar{X})^4}{s^4} - 3$$

where:
s = sample standard deviation

For a sample size greater than 100, a sample excess kurtosis of greater than 1.0 would be considered unusually high. Most equity return series have been found to be leptokurtic.

Odds for an event

$$P(E) = \frac{a}{(a+b)}$$

Where the odds for are given as ‘a to b’, then:

Odds for an event

$$P(E) = \frac{b}{(a+b)}$$

Where the odds *against* are given as ‘a to b’, then:

Conditional Probabilities

$$P(A|B) = \frac{P(AB)}{P(B)} \text{ given that } P(B) \neq 0$$

Multiplication Rule for Probabilities

$$P(AB) = P(A \mid B) \times P(B)$$

Addition Rule for Probabilities

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

For Independant Events

$P(A|B) = P(A)$, or equivalently, $P(B|A) = P(B)$

$P(A \text{ or } B) = P(A) + P(B) - P(AB)$
$P(A \text{ and } B) = P(A) \times P(B)$

The Total Probability Rule

$P(A) = P(AS) + P(AS^c)$

$P(A) = P(A|S) \times P(S) + P(A|S^c) \times P(S^c)$

The Total Probability Rule for *n* Possible Scenarios

$P(A) = P(A|S_1) \times P(S_1) + P(A|S_2) \times P(S_2) + ...+ P(A|S_n) \times P(S_n)$

where the set of events $\{S_1, S_2,..., S_n\}$ is mutually exclusive and exhaustive.

Expected Value

$$E(X) = P(X_1)X_1 + P(X_2)X_2 + \ldots\ldots P(X_n)X_n$$

$$E(X) = \sum_{i=1}^{n} P(X_i)X_i$$

Where:
X_i = one of n possible outcomes.

Variance and Standard Deviation

$$\sigma^2(X) = E\{[X - E(X)]^2\}$$

$$\sigma^2(X) = \sum_{i=1}^{n} P(X_i)\,[X_i - E(X)]^2$$

The Total Probability Rule for Expected Value

1. $E(X) = E(X|S)P(S) + E(X|S^c)P(S^c)$
2. $E(X) = E(X|S_1) \times P(S_1) + E(X|S_2) \times P(S_2) + ... + E(X|S_n) \times P(S_n)$

Where:
$E(X)$ = the unconditional expected value of X
$E(X|S_1)$ = the expected value of X given Scenario 1
$P(S_1)$ = the probability of Scenario 1 occurring
The set of events $\{S_1, S_2, ..., S_n\}$ is mutually exclusive and exhaustive.

Covariance

$$Cov\,(XY) = E\{[X - E(X)][Y - E(Y)]\}$$

$$Cov\,(R_A,R_B) = E\{[R_A - E(R_A)][R_B - E(R_B)]\}$$

Correlation Coefficient

$$Corr\,(R_A,R_B) = \rho(R_A,R_B) = \frac{Cov\,(R_A,R_B)}{(\sigma_A)(\sigma_B)}$$

Expected Return on a Portfolio

$$E(R_p) = \sum_{i=1}^{N} w_i E(R_i) = w_1 E(R_1) + w_2 E(R_2) + ... + w_N E(R_N)$$

Where:

$$\text{Weight of asset } i = \frac{\text{Market value of investment } i}{\text{Market value of portfolio}}$$

Portfolio Variance

$$Var(R_p) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j Cov(R_i, R_j)$$

Variance of a 2 Asset Portfolio

$$Var(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + 2w_A w_B Cov(R_A, R_B)$$

$$Var(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + 2w_A w_B \rho(R_A, R_B)\sigma(R_A)\sigma(R_B)$$

Variance of a 3 Asset Portfolio

$$\text{Var}(R_p) = w_A^2\sigma^2(R_A) + w_B^2\sigma^2(R_B) + w_C^2\sigma^2(R_C) + 2w_Aw_B\text{Cov}(R_A,R_B) + 2w_Bw_C\text{Cov}(R_B,R_C) + 2w_Cw_A\text{Cov}(R_C,R_A)$$

Bayes' Formula

$$P\left(\text{Event} \mid \text{Information}\right) = \frac{P\left(\text{Information} \mid \text{Event}\right) \times P\left(\text{Event}\right)}{P\left(\text{Information}\right)}$$

Counting Rules

The number of different ways that the k tasks can be done equals $n_1 \times n_2 \times n_3 \times \ldots n_k$.

Combinations

$$_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!(r!)}$$

Remember: The combination formula is used when the order in which the items are assigned the labels is NOT important.

Permutations

$$_nP_r = \frac{n!}{(n-r)!}$$

Discrete uniform distribution

$F(x) = n \times p(x)$ for the *nth* observation.

Binomial Distribution

$$P(X=x) = {}_nC_x\,(p)^x\,(1-p)^{n-x}$$

where:
p = probability of success
$1 - p$ = probability of failure
$_nC_x$ = number of possible combinations of having x successes in n trials. Stated differently, it is the number of ways to choose x from n when the order does not matter.

Variance of a binomial random variable

$$\sigma_x^2 = n \times p \times (1-p)$$

The Continuous Uniform Distribution

$P(X < a), P(X > b) = 0$

$$P(x_1 \leq X \leq x_2) = \frac{x_2 - x_1}{b - a}$$

Confidence Intervals

For a random variable X that follows the normal distribution:
The 90% confidence interval is $\bar{x} - 1.65s$ to $\bar{x} + 1.65s$
The 95% confidence interval is $\bar{x} - 1.96s$ to $\bar{x} + 1.96s$
The 99% confidence interval is $\bar{x} - 2.58s$ to $\bar{x} + 2.58s$

The following probability statements can be made about normal distributions

- Approximately 50% of all observations lie in the interval $\mu \pm (2/3)\sigma$
- Approximately 68% of all observations lie in the interval $\mu \pm 1\sigma$
- Approximately 95% of all observations lie in the interval $\mu \pm 2\sigma$
- Approximately 99% of all observations lie in the interval $\mu \pm 3\sigma$

z-Score

z = (observed value - population mean)/standard deviation = $(x - \mu)/\sigma$

Roy's safety-first criterion

Minimize $P(R_P < R_T)$
where:
R_P = portfolio return
R_T = target return

Shortfall Ratio

$$\text{Shortfall ratio (SF Ratio) or z-score} = \frac{E(R_P) - R_T}{\sigma_P}$$

Continuously Compounded Returns

$EAR = e^{r_{cc}} - 1$ $\quad$ r_{cc} = continuously compounded annual rate

$$HPR_t = e^{r_{cc} \times t} - 1$$

Sampling Error

Sampling error of the mean = Sample mean - Population mean = $\bar{x} - \mu$

Standard Error of Sample Mean when Population variance is Known

$$\sigma_{\bar{x}} = \sigma / \sqrt{n}$$

where:
$\sigma_{\bar{x}}$ = the standard error of the sample mean
σ = the population standard deviation
n = the sample size

Standard Error of Sample Mean when Population variance is Not Known

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

where:
$s_{\bar{x}}$ = standard error of sample mean
s = sample standard deviation.

Confidence Intervals

Point estimate ± (reliability factor × standard error)

where:
Point estimate = value of the sample statistic that is used to estimate the population parameter
Reliability factor = a number based on the assumed distribution of the point estimate and the level of confidence for the interval (1- α).
Standard error = the standard error of the sample statistic (point estimate)

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

where:

$\bar{x}$ = The sample mean (point estimate of population mean)
$z_{\alpha/2}$ = The standard normal random variable for which the probability of an observation lying in either tail is $\alpha / 2$ (reliability factor).
$\frac{\sigma}{\sqrt{n}}$ = The standard error of the sample mean.

$$\bar{x} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}$$

where:
$\bar{x}$ = sample mean (the point estimate of the population mean)
$t_{\frac{\alpha}{2}}$ = the t-reliability factor
$\frac{s}{\sqrt{n}}$ = standard error of the sample mean
s = sample standard deviation

Test Statistic

$$\text{Test statistic} = \frac{\text{Sample statistic - Hypothesized value}}{\text{Standard error of sample statistic}}$$

Power of a Test

Power of a test = 1 - P(Type II error)

Decision Rules for Hypothesis Tests

Decision	H_0 is True	H_0 is False
Do not reject H_0	Correct decision	Incorrect decision **Type II error**
Reject H_0	Incorrect decision **Type I error** Significance level = P(Type I error)	Correct decision Power of the test = 1 - P(Type II error)

Confidence Interval

$$\left[\left(\begin{array}{c}\text{sample}\\ \text{statistic}\end{array}\right) - \left(\begin{array}{c}\text{critical}\\ \text{value}\end{array}\right)\left(\begin{array}{c}\text{standard}\\ \text{error}\end{array}\right)\right] \leq \left(\begin{array}{c}\text{population}\\ \text{parameter}\end{array}\right) \leq \left[\left(\begin{array}{c}\text{sample}\\ \text{statistic}\end{array}\right) + \left(\begin{array}{c}\text{critical}\\ \text{value}\end{array}\right)\left(\begin{array}{c}\text{standard}\\ \text{error}\end{array}\right)\right]$$

$$\bar{x} - (z_{\alpha/2})\,(s/\sqrt{n}) \leq \mu_0 \leq \bar{x} + (z_{\alpha/2})\,(s/\sqrt{n})$$

Summary

Type of test	Null hypothesis	Alternate hypothesis	Reject null if	Fail to reject null if	P-value represents
One tailed (upper tail) test	$H_0 : \mu \leq \mu_0$	$H_a : \mu > \mu_0$	Test statistic > critical value	Test statistic ≤ critical value	Probability that lies above the computed test statistic.
One tailed (lower tail) test	$H_0 : \mu \geq \mu_0$	$H_a : \mu < \mu_0$	Test statistic < critical value	Test statistic ≥ critical value	Probability that lies below the computed test statistic.
Two-tailed	$H_0 : \mu = \mu_0$	$H_a : \mu \neq \mu_0$	Test statistic < Lower critical value Test statistic > Upper critical value	Lower critical value ≤ test statistic ≤ Upper critical value	Probability that lies above the positive value of the computed test statistic *plus* the probability that lies below the negative value of the computed test statistic

t-Statistic

$$\text{t-stat} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Where:
x = sample mean
μ_0= hypothesized population mean
s = standard deviation of the sample
n = sample size

z-Statistic

$$\text{z-stat} = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Where:
x = sample mean
μ_0= hypothesized population mean
σ = standard deviation of the population
n = sample size

$$\text{z-stat} = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Where:
x = sample mean
μ_0= hypothesized population mean
s = standard deviation of the sample
n = sample size

Tests for Means when Population Variances are Assumed Equal

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}\right)^{1/2}}$$

Where:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

s_1^2 = variance of the first sample

s_2^2 = variance of the second sample

n_1 = number of observations in first sample

n_2 = number of observations in second sample

degrees of freedom = $n_1 + n_2 - 2$

Tests for Means when Population Variances are Assumed Unequal

$$\text{t-stat} = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^{1/2}}$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1} + \frac{(s_2^2/n_2)^2}{n_2}}$$

Where:

s_1^2 = variance of the first sample

s_2^2 = variance of the second sample

n_1 = number of observations in first sample

n_2 = number of observations in second sample

Paired Comparisons Test

$$t = \frac{\bar{d} - \mu_{dz}}{s_{\bar{d}}}$$

Where:

$\bar{d}$ = sample mean difference

$s_{\bar{d}}$ = standard error of the mean difference $= \frac{s_d}{\sqrt{n}}$

s_d = sample standard deviation

n = the number of paired observations

Hypothesis Tests Concerning the Mean of Two Populations - Appropriate Tests

Population distribution	Relationship between samples	Assumption regarding variance	Type of test
Normal	Independent	Equal	t-test pooled variance
Normal	Independent	Unequal	t-test with variance not pooled
Normal	Dependent	N/A	t-test with paired comparisons

Chi Squared Test-Statistic

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Where:
n = sample size
s^2 = sample variance
σ_0^2 = hypothesized value for population variance

Test-Statistic for the F-Test

$$F = \frac{s_1^2}{s_2^2}$$

Where:
s_1^2 = Variance of sample drawn from Population 1
s_2^2 = Variance of sample drawn from Population 2

Hypothesis tests concerning the variance.

Hypothesis Test Concerning	Appropriate test statistic
Variance of a single, normally distributed population	Chi-square stat
Equality of variance of two independent, normally distributed populations	F-stat

Setting Price Targets with Head and Shoulders Patterns

Price target = Neckline - (Head - Neckline)

Setting Price Targets for Inverse Head and Shoulders Patterns

Price target = Neckline + (Neckline - Head)

Momentum or Rate of Change Oscillator

$M = (V - Vx) \times 100$

where:
M = momentum oscillator value
V = last closing price
V*x* = closing price *x* days ago, typically 10 days

Relative Strength Index

$$\text{RSI} = 100 - \frac{100}{1 + \text{RS}}$$

$$\text{where RS} = \frac{\Sigma\ (\text{Up changes for the period under consideration})}{\Sigma\ (|\text{Down changes for the period under consideration}|)}$$

Stochastic Oscillator

$$\%K = 100\left(\frac{C - L14}{H14 - L14}\right)$$

where:
C = last closing price
L14 = lowest price in last 14 days
H14 = highest price in last 14 days

%D (signal line) = Average of the last three %K values calculated daily.

Short Interest ratio

$$\text{Short interest ratio} = \frac{\text{Short interest}}{\text{Average daily trading volume}}$$

Arms Index

$$\text{Arms Index} = \frac{\text{Number of advancing issues / Number of declining issues}}{\text{Volume of advancing issues / Volume of declining issues}}$$

Economics

Demand and Supply Analysis: Introduction

The demand function captures the effect of all these factors on demand for a good.

$$\text{Demand function: } QD_x = f(P_x, I, P_y, \ldots) \quad \ldots \text{(Equation 1)}$$

Equation 1 is read as "the quantity demanded of Good X (QD_X) depends on the price of Good X (P_X), consumers' incomes (I) and the price of Good Y (P_Y), etc."

The supply function can be expressed as:

$$\text{Supply function: } QS_x = f(P_x, W, \ldots) \quad \ldots \text{(Equation 5)}$$

The own-price elasticity of demand is calculated as:

$$ED_{Px} = \frac{\%\Delta QD_x}{\%\Delta P_x} \quad \ldots \text{(Equation 16)}$$

If we express the percentage change in X as the change in X divided by the value of X, Equation 16 can be expanded to the following form:

$$ED_{Px} = \frac{\%\Delta QD_x}{\%\Delta P_x} = \frac{\Delta QD_x / QD_x}{\Delta P_x / P_x} = \left(\frac{\Delta QD_x}{\Delta P_x}\right)\left(\frac{P_x}{QD_x}\right) \quad \ldots \text{(Equation 17)}$$

Slope of demand function.

Coefficient on own-price in market demand function

Arc elasticity is calculated as:

$$E_P = \frac{\%\text{ change in quantity demanded}}{\%\text{ change in price}} = \frac{\%\,\Delta\,Q_d}{\%\,\Delta\,P} = \frac{\dfrac{(Q_0 - Q_1)}{(Q_0 + Q_1)/2} \times 100}{\dfrac{(P_0 - P_1)}{(P_0 + P_1)/2} \times 100}$$

Income Elasticity of Demand

Income elasticity of demand measures the responsiveness of demand for a particular good to a change in income, holding all other things constant.

Same as coefficient on I in market demand function (Equation 11)

$$ED_I = \frac{\%\Delta QD_x}{\%\Delta I} = \frac{\Delta QD_x / QD_x}{\Delta I / I} = \left(\frac{\Delta QD_x}{\Delta I}\right)\left(\frac{I}{QD_x}\right) \quad \text{… (Equation 18)}$$

$$E_I = \frac{\text{\% change in quantity demanded}}{\text{\% change in income}}$$

Cross-Price Elasticity of Demand

Cross elasticity of demand measures the responsiveness of demand for a particular good to a change in price of *another* good, holding all other things constant.

Same as coefficient on P_Y in market demand function (Equation 11)

$$ED_{Py} = \frac{\%\Delta QD_x}{\%\Delta P_y} = \frac{\Delta QD_x / QD_x}{\Delta P_y / P_y} = \left(\frac{\Delta QD_x}{\Delta P_y}\right)\left(\frac{P_y}{QD_x}\right) \quad \text{… (Equation 19)}$$

$$E_C = \frac{\text{\% change in quantity demanded}}{\text{\% change in price of substitute or complement}}$$

Demand And Supply Analysis: Consumer Demand

he Utility Function

In general a utility function can be represented as:

$$U = f(Q_{x_1}, Q_{x_2},..., Q_{x_n})$$

Demand and Supply Analysis: The Firm

Accounting Profit

Accounting profit (loss) = Total revenue – Total accounting costs.

Economic Profit

Economic profit (also known as abnormal profit or supernormal profit) is calculated as:

Economic profit = Total revenue – Total economic costs

Economic profit = Total revenue – (Explicit costs + Implicit costs)

Economic profit = Accounting profit – Total implicit opportunity costs

Normal Profit

Normal profit = Accounting profit - Economic profit

Total, Average and Marginal Revenue

Table 2: Summary of Revenue Terms [2]

Revenue	Calculation
Total revenue (TR)	Price times quantity (P × Q), or the sum of individual units sold times their respective prices; Σ(Pi × Qi)
Average revenue (AR)	Total revenue divided by quantity; (TR / Q)
Marginal revenue (MR)	Change in total revenue divided by change in quantity; (ΔTR / ΔQ)

Total, Average, Marginal, Fixed and Variable Costs

Table 5: Summary of Cost Terms [3]

Costs	Calculation
Total fixed cost (TFC)	Sum of all fixed expenses; here defined to include all opportunity costs
Total variable cost (TVC)	Sum of all variable expenses, or per unit variable cost times quantity; (per unit VC × Q)
Total costs (TC)	Total fixed cost plus total variable cost; (TFC + TVC)
Average fixed cost (AFC)	Total fixed cost divided by quantity; (TFC / Q)
Average variable cost (AVC)	Total variable cost divided by quantity; (TVC / Q)
Average total cost (ATC)	Total cost divided by quantity; (TC / Q) or (AFC + AVC)
Marginal cost (MC)	Change in total cost divided by change in quantity; (ΔTC / ΔQ)

Marginal revenue product (MRP) of labor is calculated as:

MRP of labor = Change in total revenue / Change in quantity of labor

For a firm in perfect competition, MRP of labor equals the MP of the last unit of labor times the price of the output unit.

MRP = Marginal product * Product price

A profit-maximizing firm will hire more labor until:

$$MRP_{Labor} = Price_{Labor}$$

Profits are maximized when:

$$\frac{MRP_1}{\text{Price of input 1}} = \ldots = \frac{MRP_n}{\text{Price of input n}}$$

[2] Exhibit 3, pg 106, Volume 2, CFA Program Curriculum 2012

The Firm And Market Structures

The relationship between MR and price elasticity can be expressed as:

$$MR = P[1 - (1/E_P)]$$

In a monopoly, MC = MR so:

$$P[1 - (1/E_P)] = MC$$

N-firm concentration ratio: Simply computes the aggregate market share of the N largest firms in the industry. The ratio will equal 0 for perfect competition and 100 for a monopoly.

Herfindahl-Hirschman Index (HHI): Adds up the squares of the market shares of each of the largest N companies in the market. The HHI equals 1 for a monopoly. If there are M firms in the industry with equal market shares, the HHI will equal 1/M.

Aggregate Output, Price, And Economic Growth

Nominal GDP refers to the value of goods and services included in GDP measured at **current prices**.

$$\text{Nominal GDP} = \text{Quantity produced in Year t} \times \text{Prices in Year t}$$

Real GDP refers to the value of goods and services included in GDP measured at **base-year prices**.

$$\text{Real GDP} = \text{Quantity produced in Year t} \times \text{Base-year prices}$$

GDP Deflator

$$\text{GDP deflator} = \frac{\text{Value of current year output at current year prices}}{\text{Value of current year output at base year prices}} \times 100$$

$$\text{GDP deflator} = \frac{\text{Nominal GDP}}{\text{Real GDP}} \times 100$$

The Components of GDP

Based on the expenditure approach, GDP may be calculated as:

$$GDP = C + I + G + (X - M)$$

C = Consumer spending on final goods and services
I = Gross private domestic investment, which includes business investment in capital goods (e.g. plant and equipment) and changes in inventory (inventory investment)
G = Government spending on final goods and services
X = Exports
M = Imports

Expenditure Approach

Under the expenditure approach, GDP at market prices may be calculated as:

This equation is just a breakdown of the expression for GDP we stated in the previous LOS, i.e. GDP = C + I + G + (X – M).

GDP = Consumer spending on goods and services
+ Business gross fixed investment
+ Change in inventories
+ Government spending on goods and services
+ Government gross fixed investment
+ Exports – Imports
+ Statistical discrepancy

Income Approach

Under the income approach, GDP at market prices may be calculated as:

GDP = National income + Capital consumption allowance
+ Statistical discrepancy … (Equation 1)

National income equals the sum of incomes received by all factors of production used to generate final output. It includes:

- Employee compensation
- Corporate and government enterprise profits before taxes, which includes:
  - Dividends paid to households
  - Corporate profits retained by businesses
  - Corporate taxes paid to the government
- Interest income
- Rent and unincorporated business net income (proprietor's income): Amounts earned by unincorporated proprietors and farm operators, who run their own businesses.
- Indirect business taxes less subsidies: This amount reflects taxes and subsidies that are included in the final price of a good or service, and therefore represents the portion of national income that is directly paid to the government.

The **capital consumption allowance (CCA)** accounts for the wear and tear or depreciation that occurs in capital stock during the production process. It represents the amount that must be reinvested by the company in the business to maintain current productivity levels. You should think of profits + CCA as the amount earned by capital.

Personal income = National income
– Indirect business taxes
– Corporate income taxes
– Undistributed corporate profits
+ Transfer payments … (Equation 2)

Personal disposable income = Personal income – Personal taxes … (Equation 3)

Personal disposable income = Household consumption + Household saving … (Equation 4)

Household saving = Personal disposable income
– Consumption expenditures
– Interest paid by consumers to businesses
– Personal transfer payments to foreigners … (Equation 5)

Business sector saving = Undistributed corporate profits
+ Capital consumption allowance … (Equation 6)

GDP = Household consumption + Total private sector saving + Net taxes

The equality of expenditure and income

S = I + (G – T) + (X – M) … (Equation 7)

The IS Curve (Relationship between Income and the Real Interest Rate)

Disposable income = GDP – Business saving – Net taxes

S – I = (G – T) + (X – M) … (Equation 7)

The LM Curve

Quantity theory of money: $MV = PY$

The quantity theory equation can also be written as:

M/P and $M_D/P = kY$

where :

$k = I/V$
M = Nominal money supply
M_D = Nominal money demand
M_D/P is referred to as real money demand and M/P is real money supply.

Equilibrium in the money market requires that money supply and money demand be equal.

Money market equilibrium: $M/P = RM_D$

Solow (neoclassical) growth model

$Y = AF(L,K)$

Where:

Y = Aggregate output
L = Quantity of labor
K = Quantity of capital
A = Technological knowledge or total factor productivity (TFP)

Growth accounting equation

Growth in potential GDP = Growth in technology + W_L(Growth in labor) + W_K(Growth in capital)

Growth in per capital potential GDP = Growth in technology + W_K(Growth in capital-labor ratio)

Measures of Sustainable Growth

Labor productivity = Real GDP/ Aggregate hours

Potential GDP = Aggregate hours × Labor productivity

This equation can be expressed in terms of growth rates as:

Potential GDP growth rate = Long-term growth rate of labor force + Long-term labor productivity growth rate

Understanding Business Cycles

Unit labor cost (ULC) is calculated as:

ULC = W/O

Where:

O = Output per hour per worker
W = Total labor compensation per hour per worker

Monetary And Fiscal Policy

Required reserve ratio = Required reserves / Total deposits

Money multiplier = 1/ (Reserve requirement)

The Fischer effect states that the nominal interest rate (R_N) reflects the real interest rate (R_R) and the expected rate of inflation (Π^e).

$$R_N = R_R + \Pi^e$$

The Fiscal Multiplier

Ignoring taxes, the multiplier can also be calculated as:

- 1/(1-MPC) = 1/(1-0.9) = 10

Assuming taxes, the multiplier can also be calculated as:

$$\frac{1}{[1 - MPC(1-t)]}$$

International Trade And Capital Flows

Balance of Payment Components

A country's balance of payments is composed of three main accounts.

- The current account balance largely reflects trade in goods and services.
- The capital account balance mainly consists of capital transfers and net sales of non-produced, non-financial assets.
- The financial account measures net capital flows based on sales and purchases of domestic and foreign financial assets.

Currency Exchange Rates

The real exchange rate may be calculated as:

$$\text{Real exchange rate}_{DC/FC} = S_{DC/FC} \times (P_{FC} / P_{DC})$$

where:
$S_{DC/FC}$ = Nominal spot exchange rate
P_{FC} = Foreign price level quoted in terms of the foreign currency
P_{DC} = Domestic price level quoted in terms of the domestic currency

The forward rate may be calculated as:

$$F_{DC/FC} = \frac{1}{S_{FC/DC}} \times \frac{(1 + r_{DC})}{(1 + r_{FC})} \text{ or } F_{DC/FC} = S_{DC/FC} \times \frac{(1 + r_{DC})}{(1 + r_{FC})}$$

This version of the formula is perhaps easiest to remember because it contains the DC term in numerator for all three components: $F_{DC/FC}$, $S_{DC/FC}$ and $(1 + r_{DC})$

Forward rates are sometimes interpreted as expected future spot rates.

$$F_t = S_{t+1}$$

$$\frac{(S_{t+1})}{S} - 1 = \Delta\%S(DC/FC)_{t+1} = \frac{(r_{DC} - r_{FC})}{(1 + r_{FC})}$$

Exchange Rates and the Trade Balance

The Elasticities Approach

$$\text{Marshall-Lerner condition: } \omega_X \varepsilon_X + \omega_M(\varepsilon_M - 1) > 0$$

Where:
ω_X = Share of exports in total trade
ω_M = Share of imports in total trade
ε_X = Price elasticity of demand for exports
ε_M = Price elasticity of demand for imports

Financial Reporting and Analysis

Exhibit 1: Accruals [2]

Cash movement in the same period as accounting recognition

↓

Settled transaction-no accrual entry needed

Cash Movement prior to Accounting Recognition

Unearned (deferred) Revenue

Originating Entry
Record cash receipt and establish a liability
Adjusting Entry
Reduce the liability and recognize revenue

Prepaid Expense

Originating Entry
Record cash payment and establish an asset
Adjusting Entry
Reduce the asset and recognize the expense

Cash Movement after Accounting Recognition

Unbilled (accrued) Revenue

Originating Entry
Record revenue and establish an asset
Adjusting Entry
When billing occurs, reduce unbilled revenue and increase accounts receivable. When cash is collected, eliminate the receivable

Accrued Expenses

Originating Entry
Establish a liability and record an expense
Adjusting Entry
Reduce the liability as cash is paid

[2] Exhibit 10, pg 72, Vol 3, CFA Program Curriculum 2012

Basic EPS

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of shares outstanding}}$$

Diluted EPS

$$\text{Diluted EPS} = \frac{\left[\text{Net income} - \text{Preferred dividends}\right] + \text{Convertible preferred dividends} + \left[\text{Convertible debt interest} \times (1 - t)\right]}{\text{Weighted average shares} + \text{Shares from conversion of convertible preferred shares} + \text{Shares from conversion of convertible debt} + \text{Shares issuable from stock options}}$$

Comprehensive Income

Net income + Other comprehensive income = Comprehensive income

Gains and Losses on Marketable Securities

	Held-to-Maturity Securities	Available-for-sale Securities	Trading Securities
Balance Sheet	Reported at cost or amortized cost.	Reported at fair value. Unrealized gains or losses due to changes in market values are reported in other comprehensive income within owners' equity.	Reported at fair value.
Items recognized on the income statement	Interest income Realized gains and losses.	Dividend income. Interest income. Realized gains and losses.	Dividend income. Interest income. Realized gains and losses. Unrealized gains and losses due to changes in market values.

Cash Flow Classification under U.S. GAAP

CFO

Inflows	Outflows
Cash collected from customers. Interest and dividends received. Proceeds from sale of securities held for trading.	Cash paid to employees. Cash paid to suppliers. Cash paid for other expenses. Cash used to purchase trading securities. Interest paid. Taxes paid.

CFI

Inflows	Outflows
Sale proceeds from fixed assets. Sale proceeds from long-term investments.	Purchase of fixed assets. Cash used to acquire LT investment securities.

CFF

Inflows	Outflows
Proceeds from debt issuance. Proceeds from issuance of equity instruments.	Repayment of LT debt. Payments made to repurchase stock. Dividends payments.

Cash Flow Statements under IFRS and U.S. GAAP

	IFRS	U.S. GAAP
Classification of Cash Flows		
Interest and dividends received	CFO or CFI	CFO
Interest paid	CFO or CFF	CFO
Dividend paid	CFO or CFF	CFF
Dividends received	CFO or CFI	CFO
Taxes paid	CFO, but part of the tax can be categorized as CFI or CFF if it is clear that the tax arose from investing or financing activities.	CFO
Bank overdrafts	Included as a part of cash equivalents.	Not considered a part of cash equivalents and included in CFF.
Presentation Format		
CFO (No difference in CFI and CFF presentation)	Direct or indirect method. The former is preferred.	Direct or indirect method. The former is preferred. However, if the direct method is used, a reconciliation of net income and CFO must be included.
Disclosures		
	Taxes paid should be presented separately on the cash flow statement.	If taxes and interest paid are not explicitly stated on the cash flow statement, details can be provided in footnotes.

Free Cash Flow to the Firm

FCFF = NI + NCC + [Int * (1 – tax rate)] – FCInv – WCInv

FCFF = CFO + [Int * (1 – tax rate)] – FCInv

Free Cash Flow to Equity

FCFE = CFO - FCInv + Net borrowing

Inventory Turnover

Inventory turnover $= \frac{\text{Cost of goods sold}}{\text{Average inventory}}$

Days of Inventory on Hand

Days of inventory on hand (DOH) $= \frac{365}{\text{Inventory turnover}}$

Receivables Turnover

Receivables turnover $= \frac{\text{Revenue}}{\text{Average receivables}}$

Days of Sales Outstanding

Days of sales outstanding (DSO) $= \frac{365}{\text{Receivables turnover}}$

Payables Turnover

Payables turnover $= \frac{\text{Purchases}}{\text{Average trade payables}}$

Number of Days of Payables

Number of days of payables $= \frac{365}{\text{Payables turnover}}$

Working Capital Turnover

Working capital turnover $= \frac{\text{Revenue}}{\text{Average working capital}}$

Fixed Asset Turnover

Fixed asset turnover $= \frac{\text{Revenue}}{\text{Average fixed assets}}$

Total Asset Turnover

Total Asset Turnover $= \frac{\text{Revenue}}{\text{Average total assets}}$

Current Ratio

Current ratio $= \frac{\text{Current assets}}{\text{Current liabilities}}$

Quick Ratio

Quick ratio $= \frac{\text{Cash + Short-term marketable investments + Receivables}}{\text{Current liabilities}}$

Cash Ratio

Cash ratio $= \frac{\text{Cash + Short-term marketable investments}}{\text{Current liabilities}}$

Defensive Interval Ratio

Defensive interval ratio $= \frac{\text{Cash + Short-term marketable investments + Receivables}}{\text{Daily cash expenditures}}$

Cash Conversion Cycle

Cash conversion cycle = DSO + DOH – Number of days of payables

Debt-to-Assets Ratio

Debt-to-assets ratio $= \frac{\text{Total debt}}{\text{Total assets}}$

Debt-to-Capital Ratio

Debt-to-capital ratio $= \frac{\text{Total debt}}{\text{Total debt + Shareholders' equity}}$

Debt-to-Equity Ratio

Debt-to-equity ratio $= \frac{\text{Total debt}}{\text{Shareholders' equity}}$

Financial Leverage Ratio

Financial leverage ratio $= \frac{\text{Average total assets}}{\text{Average total equity}}$

Interest Coverage Ratio

Interest coverage ratio $= \frac{\text{EBIT}}{\text{Interest payments}}$

Fixed Charge Coverage Ratio

Fixed charge coverage ratio $= \frac{\text{EBIT + Lease payments}}{\text{Interest payments + Lease payments}}$

Gross Profit Margin

Gross profit margin $= \frac{\text{Gross profit}}{\text{Revenue}}$

Operating Profit Margin

$$\textbf{Operating profit margin} = \frac{\text{Operating profit}}{\text{Revenue}}$$

Pretax Margin

$$\textbf{Pretax margin} = \frac{\text{EBT (earnings before tax, but after interest)}}{\text{Revenue}}$$

Net Profit Margin

$$\textbf{Net profit margin} = \frac{\text{Net profit}}{\text{Revenue}}$$

Return on Assets

$$\textbf{ROA} = \frac{\text{Net income}}{\text{Average total assets}}$$

$$\textbf{Adjusted ROA} = \frac{\text{Net income + Interest expense (1 – Tax rate)}}{\text{Average total assets}}$$

$$\textbf{Operating ROA} = \frac{\text{Operating income or EBIT}}{\text{Average total assets}}$$

Return on Total Capital

$$\textbf{Return on total capital} = \frac{\text{EBIT}}{\text{Short-term debt + Long-term debt + Equity}}$$

Return on Equity

$$\textbf{Return on equity} = \frac{\text{Net income}}{\text{Average total equity}}$$

Return on Common Equity

$$\textbf{Return on common equity} = \frac{\text{Net income – Preferred dividends}}{\text{Average common equity}}$$

DuPont Decomposition of ROE

$$\textbf{ROE} = \frac{\text{Net income}}{\text{Average shareholders' equity}}$$

2-Way Dupont Decomposition

$$\textbf{ROE} = \underbrace{\frac{\text{Net income}}{\text{Average total assets}}}_{\text{ROA}} \times \underbrace{\frac{\text{Average total assets}}{\text{Average shareholder's equity}}}_{\text{Leverage}}$$

3-Way Dupont Decomposition

$$\textbf{ROE} = \underbrace{\frac{\text{Net income}}{\text{Revenue}}}_{\text{Net profit margin}} \times \underbrace{\frac{\text{Revenue}}{\text{Average total assets}}}_{\text{Asset turnover}} \times \underbrace{\frac{\text{Average total assets}}{\text{Average shareholders' equity}}}_{\text{Leverage}}$$

5-Way Dupont Decomposition

$$\text{ROE} = \underbrace{\frac{\text{Net income}}{\text{EBT}}}_{\text{Tax burden}} \times \overbrace{\frac{\text{EBT}}{\text{EBIT}}}^{\text{Interest burden}} \times \underbrace{\frac{\text{EBIT}}{\text{Revenue}}}_{\text{EBIT margin}} \times \overbrace{\frac{\text{Revenue}}{\text{Average total assets}}}^{\text{Asset turnover}} \times \underbrace{\frac{\text{Average total assets}}{\text{Avg. shareholders' equity}}}_{\text{Leverage}}$$

Price- to-Earnings Ratio

$$\textbf{P/E} = \frac{\text{Price per share}}{\text{Earnings per share}}$$

Price to Cash Flow

$$\textbf{P/CF} = \frac{\text{Price per share}}{\text{Cash flow per share}}$$

Price to Sales

$$\textbf{P/S} = \frac{\text{Price per share}}{\text{Sales per share}}$$

Price to Book Value

$$\textbf{P/BV} = \frac{\text{Price per share}}{\text{Book value per share}}$$

Per Share Ratios

$$\textbf{Cash flow per share} = \frac{\text{Cash flow from operations}}{\text{Average number of shares outstanding}}$$

$$\textbf{EBITDA per share} = \frac{\text{EBITDA}}{\text{Average number of shares outstanding}}$$

$$\textbf{Dividends per share} = \frac{\text{Common dividends declared}}{\text{Weighted average number of ordinary shares}}$$

Dividend Payout Ratio

$$\textbf{Dividend payout ratio} = \frac{\text{Common share dividends}}{\text{Net income attributable to common shares}}$$

Retention Rate

$$\textbf{Retention Rate} = \frac{\text{Net income attributable to common shares} - \text{Common share dividends}}{\text{Net income attributable to common shares}}$$

Growth Rate

Sustainable growth rate = Retention rate × ROE

LIFO versus FIFO (with rising prices and stable inventory levels.)

LIFO versus FIFO when Prices are Rising

	LIFO	FIFO
COGS	Higher	Lower
Income before taxes	Lower	Higher
Income taxes	Lower	Higher
Net income	Lower	Higher
Cash flow	Higher	Lower
EI	Lower	Higher
Working capital	Lower	Higher

Type of Ratio	Effect on Numerator	Effect on Denominator	Effect on Ratio
Profitability ratios. NP and GP margins	Income is lower under LIFO because COGS is higher	Sales are the same under both.	Lower under LIFO.
Debt to equity	Same debt levels	Lower equity under LIFO	Higher under LIFO
Current ratio	Current assets are lower under LIFO because EI is lower.	Current liabilities are the same.	Lower under LIFO
Quick ratio	Assets are higher as a result of lower taxes paid	Current liabilities are the same	Higher under LIFO
Inventory turnover	COGS is higher under LIFO	Average inventory is lower under LIFO	Higher under LIFO
Total asset turnover	Sales are the same	Lower total assets under LIFO	Higher under LIFO

Financial Statement Effects of Capitalizing versus Expensing

	Effect on Financial Statements
Initially when the cost is capitalized	• Noncurrent assets *increase*. • Cash flow from investing activities *decreases*.
In future periods when the asset is depreciated or amortized	• Noncurrent assets *decrease*. • Net income *decreases*. • Retained earnings *decrease*. • Equity *decreases*.
When the cost is expensed	• Net income *decreases* by the entire after-tax amount of the cost. • No related asset is recorded on the balance sheet and therefore, no depreciation or amortization expense is charged in future periods. • Operating cash flow *decreases*. • Expensed costs have no financial statement impact in future years.

	Capitalizing	Expensing
Net income (first year)	Higher	Lower
Net income (future years)	Lower	Higher
Total assets	Higher	Lower
Shareholders' equity	Higher	Lower
Cash flow from operations	Higher	Lower
Cash flow from investing	Lower	Higher
Income variability	Lower	Higher
Debt to equity	Lower	Higher

Straight Line Depriciation

$$\text{Depreciation expense} = \frac{\text{Original cost - Salvage value}}{\text{Depreciable life}}$$

Accelerated Depriciation

$$\text{DDB depreciation in Year X} = \frac{2}{\text{Depreciable life}} \times \text{Book value at the beginning of Year X}$$

Estimated Useful Life

$$\text{Estimated useful life} = \frac{\text{Gross investment in fixed assets}}{\text{Annual depreciation expense}}$$

Average Cost of Asset

$$\text{Average age of asset} = \frac{\text{Accumulated depreciation}}{\text{Annual depreciation expense}}$$

Remaining Useful Life

$$\text{Remaining useful life} = \frac{\text{Net investment in fixed assets}}{\text{Annual depreciation expense}}$$

Treatment of Temporary Differences

Balance Sheet Item	Carrying value vs. tax base	Results in…
Asset	Carrying amount is greater.	DTL
Asset	Tax base is greater.	DTA
Liability	Carrying amount is greater.	DTA
Liabi lity	Tax base is greater.	DTL

Income Tax Accounting under IFRS versus U.S. GAAP

	IFRS	U.S. GAAP
ISSUE SPECIFIC TREATMENTS		
Revaluation of fixed assets and intangible assets.	Recognized in equity as deferred taxes.	Revaluation is prohibited.
Treatment of undistributed profit from investment in subsidiaries.	Recognized as deferred taxes except when the parent company is able to control the distribution of profits and it is probable that temporary differences will not reverse in future.	No recognition of deferred taxes for foreign subsidiaries that fulfill indefinite reversal criteria. No recognition of deferred taxes for domestic subsidiaries when amounts are tax-free.
Treatment of undistributed profit from investments in joint ventures.	Recognized as deferred taxes except when the investor controls the sharing of profits and it is probable that there will be no reversal of temporary differences in future.	No recognition of deferred taxes for foreign corporate joint ventures that fulfill indefinite reversal criteria.
Treatment of undistributed profit from investments in associates.	Recognized as deferred taxes except when the investor controls the sharing of profits and it is probable that there will be no reversal of temporary differences in future.	Deferred taxes are recognized from temporary differences.
DEFERRED TAX MEASUREMENT		
Tax rates.	Tax rates and tax laws enacted or substantively enacted.	Only enacted tax rates and tax laws are used.
Deferred tax asset recognition.	Recognized if it is probable that sufficient taxable profit will be available in the future.	Deferred tax assets are recognized in full and then reduced by a valuation allowance if it is likely that they will not be realized.
DEFERRED TAX PRESENTATION		
Offsetting of deferred tax assets and liabilities.	Offsetting allowed only if the entity has right to legally enforce it and the balance is related to a tax levied by the same authority.	Same as in IFRS.
Balance sheet classification.	Classified on balance sheet as net noncurrent with supplementary disclosures.	Classified as either current or noncurrent based on classification of underlying asset and liability.

Effective Tax rate

$$\text{Effective tax rate} = \frac{\text{Income tax expense}}{\text{Pretax income}}$$

Income Tax Expense

Income tax expense = Taxes Payable + Change in DTL - Change in DTA

Income Statement Effects of Lease Classification

Income Statement Item	Finance Lease	Operating Lease
Operating expenses	Lower	Higher
Nonoperating expenses	Higher	Lower
EBIT (operating income)	Higher	Lower
Total expenses- early years	Higher	Lower
Total expenses- later years	Lower	Higher
Net income- early years	Lower	Higher
Net income- later years	Higher	Lower

Balance Sheet Effects of Lease Classification

Balance Sheet Item	Capital Lease	Operating Lease
Assets	Higher	Lower
Current liabilities	Higher	Lower
Long term liabilities	Higher	Lower
Total cash	Same	Same

Cash Flow Effects of Lease Classification

CF Item	Capital Lease	Operating Lease
CFO	Higher	Lower
CFF	Lower	Higher
Total cash flow	Same	Same

Impact of Lease Classification on Financial Ratios

Ratio	Numerator under Finance Lease	Denominator under Finance Lease	Effect on Ratio	Ratio Better or Worse under Finance Lease
Asset turnover	Sales- same	Assets- higher	Lower	Worse
Return on assets*	Net income lower in early years	Assets- higher	Lower	Worse
Current ratio	Current assets- same	Current liabilities- higher	Lower	Worse
Leverage ratios (D/E and D/A)	Debt- higher	Equity same. Assets higher	Higher	Worse
Return on equity*	Net income lower in early years	Equity same	Lower	Worse

* In early years of the lease agreement.

Financial Statement Effects of Lease Classification from Lessor's Perspective

	Financing Lease	Operating Lease
Total net income	Same	Same
Net income (early years)	Higher	Lower
Taxes (early years)	Higher	Lower
Total CFO	Lower	Higher
Total CFI	Higher	Lower
Total cash flow	Same	Same

Definitions of Commonly Used Solvency Ratios

Solvency Ratios	Description	Numerator	Denominator
Leverage Ratios			
Debt-to-assets ratio	Expresses the percentage of total assets financed by debt	Total debt	Total assets
Debt-to-capital ratio	Measures the percentage of a company's total capital (debt + equity) financed by debt.	Total debt	Total debt + Total shareholders' equity
Debt-to-equity ratio	Measures the amount of debt financing relative to equity financing	Total debt	Total shareholders' equity
Financial leverage ratio	Measures the amount of total assets supported by one money unit of equity.	Average total assets	Average shareholders' equity
Coverage Ratios			
Interest coverage ratio	Measures the number of times a company's EBIT could cover its interest payments.	EBIT	Interest payments
Fixed charge coverage ratio	Measures the number of times a company's earnings (before interest, taxes and lease payments) can cover the company's interest and lease payments.	EBIT + Lease payments	Interest payments + Lease payments

Adjustments related to inventory:

$$EI_{FIFO} = EI_{LIFO} + LR$$

where
LR = LIFO Reserve

$$COGS_{FIFO} = COGS_{LIFO} - (\text{Change in LR during the year})$$

Net income after tax under FIFO will be greater than LIFO net income after tax by:

Change in LIFO Reserve × (1 - Tax rate)

When converting from LIFO to FIFO assuming rising prices:

Equity (retained earnings) increase by:

LIFO Reserve × (1 - Tax rate)

Liabilities (deferred taxes) increase by:

LIFO Reserve × (Tax rate)

Current assets (inventory) increase by:

LIFO Reserve

Adjustments related to property, plant and equipment:

$$\frac{\text{Gross investment in fixed assets}}{\text{Annual depreciation expense}} = \frac{\text{Accumulated depreciation}}{\text{Annual depreciation expense}} + \frac{\text{Net investment in fixed assets}}{\text{Annual depreciation expense}}$$

Estimated useful or depreciable life

The historical cost of an asset divided by its useful life equals annual depreciation expense under the straight line method. Therefore, the historical cost divided by annual depreciation expense equals the estimated useful life.

Average age of asset

Annual depreciation expense times the number of years that the asset has been in use equals accumulated depreciation. Therefore, accumulated depreciation divided by annual depreciation equals the average age of the asset.

Remaining useful life

The book value of the asset divided by annual depreciation expense equals the number of years the asset has remaining in its useful life.

Categories of Marketable Securities and Accounting Treatment

Classification	Balance Sheet Value	Unrealized and Realized Gains and Losses	Income (Interest & Dividends)
Held-to-maturity	Amortized cost (Par value +/- unamortized premium/ discount).	*Unrealized:* Not reported *Realized:* Recognized on income statement.	Recognized on income statement.
Held-for-trading	Fair Value.	*Unrealized:* Recognized on income statement. *Realized:* Recognized on income statement.	Recognized on income statement.
Available-for-sale	Fair Value.	*Unrealized:* Recognized in other comprehensive income. *Realized:* Recognized on income statement.	Recognized on income statement.

Inventory Accounting under IFRS versus U.S. GAAP

	Balance Sheet	Permitted Cost Recognition Methods	Changes in Balance Sheet Value
U.S. GAAP	Lower of cost or market.	▪ LIFO. ▪ FIFO. ▪ Weighted average cost.	Permits inventory write downs, but not reversal of write downs.
IFRS	Lower of cost or net realizable value.	▪ FIFO. ▪ Weighted Average Cost.	Permits inventory write downs, and also reversals of write downs.

Property, Plant and Equipment

	Balance Sheet	Changes in Balance Sheet Value	Effects of Changes in Balance Sheet Value
U.S. GAAP	Cost minus accumulated depreciation.	Does not permit upward revaluation.	No effect.
IFRS	Cost minus accumulated depreciation.	Permits upward revaluation. Asset is reported at fair value at the revaluation date less accumulated depreciation following the revaluation.	The increase in the asset's value from revaluation is reported as a part of equity unless it is reversing a previously-recognized decrease in the value of the asset. A decrease in the value of the asset is reported on the income statement unless it is reversing a previously-reported upward revaluation.

Long-Term Investments

Percent Ownership	Extent of Control	Accounting Treatment
Less than 20%	No significant control	Classified as held-to-maturity, trading, or available for sale securities.
20% - 50%	Significant Influence	Equity method.
More than 50%	Significant Control	Consolidation.
Shared (joint ventures)	Joint Control	Equity method/ proportionate consolidation.

Treatment of Identifiable Intangible Assets

	Balance Sheet	Changes in Balance Sheet Value	Effects of Changes in Balance Sheet Value
U.S. GAAP	Only purchased intangibles may be recognized as assets. Internally developed items cannot be recognized as assets. Reported at cost minus accumulated amortization for assets with finite useful lives. Reported at cost minus impairment for assets with infinite useful lives.	Does not permit upward revaluation.	No effect.
IFRS	Only purchased intangibles may be recognized as assets. Internally developed items cannot be recognized as assets. Reported at cost minus accumulated amortization for assets with finite useful lives. Reported at cost minus impairment for assets with infinite useful lives.	Permits upward revaluation. Assets are reported at fair value as of the revaluation date less subsequent accumulated amortization.	An increase in value is recognized as a part of equity unless it is a reversal of a previously recognized downward revaluation. A decrease in value is recognized on the income statement unless it is a reversal of a previously recognized upward revaluation.

Long-Term Contracts

	Outcome can be reliably estimated	Outcome cannot be reliably estimated
U.S. GAAP	Percentage-of-completion method.	Completed contract method.
IFRS	Percentage-of-completion method.	Revenue is recognized to the extent that it is probable to recover contract costs. Profit is only recognized at project completion.

Corporate Finance

Net Present Value (NPV)

$$NPV = \sum_{t=1}^{n} \frac{CF_t}{(1+r)^t} - \text{Outlay}$$

where

CF_t = after-tax cash flow at time, t.

r = required rate of return for the investment. This is the firm's cost of capital adjusted for the risk inherent in the project.

Outlay = investment cash outflow at t = 0.

Internal Rate of Return (IRR)

$$\sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} = \text{Outlay} \qquad \sum_{t=1}^{n} \frac{CF_t}{(1+IRR)^t} - \text{Outlay} = 0$$

Average Accounting Rate of Return (AAR)

$$AAR = \frac{\text{Average net income}}{\text{Average book value}}$$

Profitability Index

$$PI = \frac{\text{PV of future cash flows}}{\text{Initial investment}} = 1 + \frac{NPV}{\text{Initial investment}}$$

Weighted Average Cost of Capital

$$WACC = (w_d)(r_d)(1-t) + (w_p)(r_p) + (w_e)(r_e)$$

Where:

w_d = Proportion of debt that the company uses when it raises new funds

r_d = Before-tax marginal cost of debt

t = Company's marginal tax rate

w_p = Proportion of preferred stock that the company uses when it raises new funds

r_p = Marginal cost of preferred stock

w_e = Proportion of equity that the company uses when it raises new funds

r_e = Marginal cost of equity

To Transform Debt-to-equity Ratio into a component's weight

$$\frac{D/E}{1 + D/E} = \frac{D}{D+E} = w_d$$

$$w_d + w_e = 1$$

Valuation of Bonds

$$P_0 = \left[\sum_{t=1}^{n} \frac{PMT}{\left(1+\frac{r_d}{2}\right)^t}\right] + \frac{FV}{\left(1+\frac{r_d}{2}\right)^n}$$

where:
P_0 = current market price of the bond.
PMT_t = interest payment in period t.
r_d = yield to maturity on BEY basis.
n = number of periods remaining to maturity.
FV = Par or maturity value of the bond.

Valuation of Preferred Stock

$$V_p = D_p / r_p$$

where:
V_p = current value (price) of preferred stock..
D_p = preferred stock dividend per share.
r_p = cost of preferred stock.

Required Return on a Stock

Capital Asset Pricing Model

$$r_e = R_F + \beta_i[E(R_M) - R_F]$$

where
$[E(R_M) - R_F]$ = Equity risk premium.
R_M = Expected return on the market.
β_i = Beta of stock . Beta measures the sensitivity of the stock's returns to changes in market returns.
R_F = Risk-free rate.
r_e = Expected return on stock (cost of equity)

Dividend Discount Model

$$P_0 = \frac{D_1}{r_e - g}$$

where:
P_0 = current market value of the security.
D_1= next year's dividend.
r_e = required rate of return on common equity.
g = the firm's expected constant growth rate of dividends.

Rearranging the above equation gives us a formula to calculate the required return on equity:

$$r_e = \frac{D_1}{P_0} + g$$

Sustainable Growth Rate

$$g = \left(1 - \frac{D}{EPS}\right) \times (ROE)$$

Where (1 - (D/EPS)) = Earnings retention rate

Bond Yield plus Risk Premium Approach

$$r_e = r_d + \text{ risk premium}$$

To Unlever the beta

$$\beta_{ASSET} = \beta_{EQUITY}\left[\frac{1}{1+\left((1-t)\frac{D}{E}\right)}\right]$$

To Lever the beta

$$\beta_{PROJECT} = \beta_{ASSET}\left[1+\left((1-t)\frac{D}{E}\right)\right]$$

Country Risk Premium

$$r_e = R_F + \beta\ [E(R_M) - R_F + CRP]$$

$$\text{Country risk premium} = \text{Sovereign yield spread} \times \frac{\text{Annualized standard deviation of equity index}}{\text{Annualized standard deviation of sovereign bond market in terms of the developed market currency}}$$

$$\text{Break point} = \frac{\text{Amount of capital at which a component's cost of capital changes}}{\text{Proportion of new capital raised from the component}}$$

Degree of Operating Leverage

$$DOL = \frac{\text{Percentage change in operating income}}{\text{Percentage change in units sold}}$$

$$DOL = \frac{Q \times (P - V)}{Q \times (P - V) - F}$$

where:
Q = Number of units sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
Q × (P – V) = Contribution margin (the amount that units sold contribute to covering fixed costs)
(P – V) = Contribution margin per unit

Degree of Financial Leverage

$$DFL = \frac{\text{Percentage change in net income}}{\text{Percentage change in operating income}}$$

$$DFL = \frac{[Q(P - V) - F](1 - t)}{[Q(P - V) - F - C](1 - t)} = \frac{[Q(P - V) - F]}{[Q(P - V) - F - C]}$$

where:
Q = Number of units sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
C = Fixed financial cost
t = Tax rate

Degree of Total Leverage

$$DTL = \frac{\text{Percentage change in net income}}{\text{Percentage change in the number of units sold}}$$

$$DTL = DOL \times DFL$$

$$DTL = \frac{Q \times (P - V)}{[Q(P - V) - F - C]}$$

where:
Q = Number of units produced and sold
P = Price per unit
V = Variable operating cost per unit
F = Fixed operating cost
C = Fixed financial cost

Break point

PQ = VQ + F + C

where:
P = Price per unit
Q = Number of units produced and sold
V = Variable cost per unit
F = Fixed operating costs
C = Fixed financial cost

The breakeven number of units can be calculated as:

$$Q_{BE} = \frac{F + C}{P - V}$$

Operating breakeven point

$$PQ_{OBE} = PV + F$$

$$Q_{OBE} = \frac{F}{P - V}$$

$$\text{Current Ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{Quick Ratio} = \frac{\text{cash} + \text{short term marketable investments} + \text{receivables}}{\text{Current liabilities}}$$

$$\text{Accounts receivable turnover} = \frac{\text{Credit sales}}{\text{Average receivables}}$$

$$\text{Number of days of receivables} = \frac{\text{Accounts receivable}}{\text{Average day s sales on credit}}$$

$$= \frac{\text{Accounts receivable}}{\text{Sales on credit / 365}}$$

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$\text{Number of days of inventory} = \frac{\text{Inventory}}{\text{Average day's cost of goods sold}}$$

$$= \frac{\text{Inventory}}{\text{Cost of goods sold / 365}}$$

$$\text{Payables turnover} = \frac{\text{Purchases}}{\text{Average trade payables}}$$

$$\text{Number of days of payables} = \frac{\text{Accounts payables}}{\text{Average day's purchases}}$$

$$= \frac{\text{Accounts payables}}{\text{Purchases/365}}$$

Purchases = Ending inventory + COGS - Beginning inventory

$$\text{Operating cycle} = \text{Number of days of inventory} + \text{Number of days of receivables}$$

$$\text{Net operating cycle} = \text{Number of days of inventory} + \text{Number of days of receivables} - \text{Number of days of payables}$$

$$\text{Money market yield} = \left(\frac{\text{Face value - price}}{\text{Price}}\right) \times \left(\frac{360}{\text{Days}}\right) = \text{Holding period yield} \times \left(\frac{360}{\text{Days}}\right)$$

$$\text{Bond equivalent yield} = \left(\frac{\text{Face value - price}}{\text{Price}}\right) \times \left(\frac{365}{\text{Days}}\right) = \text{Holding period yield} \times \left(\frac{365}{\text{Days}}\right)$$

$$\text{Discount basis yield} = \left(\frac{\text{Face value - price}}{\text{Face value}}\right) \times \left(\frac{360}{\text{Days}}\right) = \text{\% discount} \times \left(\frac{360}{\text{Days}}\right)$$

$$\%\ \text{Discount} = \frac{\text{Face value - Price}}{\text{Price}}$$

$$\text{Inventory turnover} = \frac{\text{Cost of goods sold}}{\text{Average inventory}}$$

$$\text{Number of days of inventory} = \frac{\text{Inventory}}{\text{Average days cost of goods sold}}$$

$$= \frac{\text{Inventory}}{\text{Cost of goods sold / 365}}$$

$$= \frac{365}{\text{Inventory turnover}}$$

$$\text{Implicit rate} = \text{Cost of trade credit} = \left(1 + \frac{\text{Discount}}{1 - \text{Discount}}\right)^{\left(365 \big/ \text{Number of days beyond discount period}\right)} - 1$$

$$\text{Number of days of payables} = \frac{\text{Accounts payable}}{\text{Average day's purchases}}$$

$$\frac{\text{Accounts payable}}{\text{Purchases / 365}} = \frac{365}{\text{Payables turnover}}$$

$$\text{Line of credit cost} = \frac{\text{Interest + Commitment fee}}{\text{Loan amount}}$$

$$\text{Banker's acceptance cost} = \frac{\text{Interest}}{\text{Net proceeds}} = \frac{\text{Interest}}{\text{Loan amount - Interest}}$$

$$\frac{\text{Interest + Dealer's comission + Backup costs}}{\text{Loan amount - Interest}}$$

$$\text{ROE} = \frac{\text{Net income}}{\text{Average total equity}}$$

$$\text{ROE} = \frac{\text{Net income}}{\text{Average total assets}} \times \frac{\text{Average total assets}}{\text{Average shareholders' equity}}$$

$$\text{ROE} = \frac{\text{Net income}}{\text{Revenue}} \times \frac{\text{Revenue}}{\text{Average total assets}} \times \frac{\text{Average total assets}}{\text{Average shareholders' equity}}$$

Portfolio Management

Holding Period Return

$$R = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}} = \text{Capital gain} + \text{Dividend yield}$$

$$= \frac{P_T + D_T}{P_0} - 1$$

where:
P_t = Price at the end of the period
P_{t-1} = Price at the beginning of the period
D_t = Dividend for the period

Holding Period Returns for more than One Period

$$R = [(1 + R_1) \times (1 + R_2) \times \ldots\ldots \times (1 + R_n)] - 1$$

where:
$R_1, R_2, \ldots, R_n$ are sub-period returns

Geometric Mean Return

$$R = \{[(1 + R_1) \times (1 + R_2) \times \ldots\ldots \times (1 + R_n)]^{1/n}\} - 1$$

Annualized Return

$$r_{annual} = (1 + r_{period})^n - 1$$

where:
r = Return on investment
n = Number of periods in a year

Portfolio Return

$$R_p = w_1R_1 + w_2R_2$$

where:
R_p = Portfolio return
w_1 = Weight of Asset 1
w_2 = Weight of Asset 2
R_1 = Return of Asset 1
R_2 = Return of Asset 2

Variance of a Single Asset

$$\sigma^2 = \frac{\sum_{t=1}^{T} (R_t - \mu)^2}{T}$$

where:
R_t = Return for the period t
T = Total number of periods
μ = Mean of T returns

Variance of a Representative Sample of the Population

$$s^2 = \frac{\sum_{t=1}^{T} (R_t - \bar{R})^2}{T-1}$$

where:
R = mean return of the sample observations
s^2 = sample variance

Standard Deviation of an Asset

$$\sigma = \sqrt{\frac{\sum_{t=1}^{T} (R_t - \mu)^2}{T}} \qquad s = \sqrt{\frac{\sum_{t=1}^{T} (R_t - \bar{R})^2}{T-1}}$$

Variance of a Portfolio of Assets

$$\sigma_P^2 = \sum_{i,j=1}^{N} w_i w_j \mathrm{Cov}(R_i, R_j)$$

$$\sigma_P^2 = \sum_{i=1}^{N} w_i^2 \mathrm{Var}(R_i) + \sum_{i,j=1,\, i \neq j}^{N} w_i w_j \mathrm{Cov}(R_i, R_j)$$

Standard Deviation of a Portfolio of Two Risky Assets

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2}} \text{ or } \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2Cov_{1,2}}$$

Utility Function

$$U = E(R) - \frac{1}{2}A\sigma^2$$

where:
U = Utility of an investment
E(R) = Expected return
σ^2 = Variance of returns
A = Additional return required by the investor to accept an additional unit of risk.

Capital Allocation Line

The CAL has an intercept of RFR and a constant slope that equals:

$$\frac{[E(R_i) - RFR]}{\sigma_i}$$

Expected Return on portfolios that lie on CML

$$E(R_p) = w_1R_f + (1 - w_1)\,E(R_m)$$

Variance of portfolios that lie on CML

$$\sigma^2 = \sqrt{w_1^2\sigma_f^2 + (1 - w_1)^2\sigma_m^2 + 2w_1(1 - w_1)Cov(R_f,R_m)}$$

Equation of CML

$$E(R_p) = R_f + \frac{E(R_m) - R_f}{\sigma_m} \times \sigma_p$$

where:
y-intercept = R_f = risk-free rate
slope = $\frac{E(R_m) - R_f}{\sigma_m}$ = market price of risk.

Systematic and Nonsystematic Risk

Total Risk = Systematic risk + Unsystematic risk

Return-Generating Models

$$E(R_i) - R_f = \sum_{j=1}^{k} \beta_{ij} E(F_j) = \beta_{i1}[E(R_m) - R_f] + \sum_{j=2}^{k} \beta_{ij} E(F_j)$$

The Market Model

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Calculation of Beta

$$\beta_i = \frac{Cov(R_i,R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i\sigma_m}{\sigma_m^2} = \frac{\rho_{i,m}\sigma_i}{\sigma_m}$$

The Capital Asset Pricing Model

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f]$$

Sharpe ratio

$$\text{Sharpe ratio} = \frac{R_p - R_f}{\sigma_p}$$

Treynor ratio

$$\text{Treynor ratio} = \frac{R_p - R_f}{\beta_p}$$

M-squared (M^2)

$$M^2 = (R_p - R_f)\frac{\sigma_m}{\sigma_p} - (R_m - R_f)$$

Jensen's alpha

$$\alpha_p = R_p - [R_f + \beta_p (R_m - R_f)]$$

Security Characteristic Line

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f)$$

EQUITY

The price at which an investor who goes long on a stock receives a margin call is calculated as:

$$P_0 \times \frac{(1 - \text{Initial margin})}{(1 - \text{Maintenance margin})}$$

The value of a price return index is calculated as follows:

$$V_{PRI} = \frac{\sum_{i=1}^{N} n_i P_i}{D}$$

where:
V_{PRI} = Value of the price return index
n_i = Number of units of constituent security i held in the index portfolio
N = Number of constituent securities in the index
P_i = Unit price of constituent security i
D = Value of the divisor

Price Return

The price return of an index can be calculated as:

$$PR_I = \frac{V_{PRI1} - V_{PRI0}}{V_{PRI0}}$$

where:
PR_I = Price return of the index portfolio (as a decimal number)
V_{PRI1} = Value of the price return index at the end of the period
V_{PRI0} = Value of the price return index at the beginning of the period

The price return of each constituent security is calculated as:

$$PR_i = \frac{P_{i1} - P_{i0}}{P_{i0}}$$

where:
PR_i = Price return of constituent security i (as a decimal number)
P_{i1} = Price of the constituent security i at the end of the period
P_{i0} = Price of the constituent security i at the beginning of the period

The price return of the index equals the weighted average price return of the constituent securities. It is calculated as:

$$PR_I = w_1PR_1 + w_2PR_2 + ...+ w_NPR_N$$

where:
PR_I = Price return of the index portfolio (as a decimal number)
PR_i = Price return of constituent security i (as a decimal number)
w_i = Weight of security i in the index portfolio
N = Number of securities in the index

Total Return

The total return of an index can be calculated as:

$$TR_I = \frac{V_{PRI1} - V_{PRI0} + Inc_I}{V_{PRI0}}$$

where:
TR_I = Total return of the index portfolio (as a decimal number)
V_{PRI1} = Value of the total return index at the end of the period
V_{PRI0} = Value of the total return index at the beginning of the period
Inc_I = Total income from all securities in the index held over the period

The total return of each constituent security is calculated as:

$$TR_i = \frac{P_{1i} - P_{0i} + Inc_i}{P_{0i}}$$

where:
TR_i = Total return of constituent security i (as a decimal number)
P_{1i} = Price of constituent security i at the end of the period
P_{0i} = Price of constituent security i at the beginning of the period
Inc_i = Total income from security i over the period

The total return of the index equals the weighted average total return of the constituent securities. It is calculated as:

$$TR_I = w_1TR_1 + w_2TR_2 + ...+ w_NTR_N$$

where:
TR_I = Total return of the index portfolio (as a decimal number)
TR_i = Total return of constituent security i (as a decimal number)
w_i = Weight of security i in the index portfolio
N = Number of securities in the index

Calculation of Index Returns over Multiple Time Periods

Given a series of price returns for an index, the value of a price return index can be calculated as:

$$V_{PRIT} = V_{PRI0}\,(1 + PR_{I1})\,(1 + PR_{I2}) \ldots (1 + PR_{IT})$$

where:
V_{PRI0} = Value of the price return index at inception
V_{PRIT} = Value of the price return index at time t
PR_{IT} = Price return (as a decimal number) on the index over the period

Similarly, the value of a total return index may be calculated as:

$$V_{TRIT} = V_{TRI0}\,(1 + TR_{I1})\,(1 + TR_{I2}) \ldots (1 + TR_{IT})$$

where:
V_{TRI0} = Value of the index at inception
V_{TRIT} = Value of the index at time t
TR_{IT} = Total return (as a decimal number) on the index over the period

Price Weighting

$$w_i^P = \frac{P_i}{\sum_{i=1}^{N} P_i}$$

Equal Weighting

$$w_i^E = \frac{1}{N}$$

where:
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
N = Number of securities in the index

Market-Capitalization Weighting

$$w_i^M = \frac{Q_i P_i}{\sum_{j=1}^{N} Q_j P_j}$$

where:
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
Q_i = Number of shares outstanding of security i
P_i = Share price of security i
N = Number of securities in the index

The float-adjusted market-capitalization weight of each constituent security is calculated as:

$$w_i^M = \frac{f_i Q_i P_i}{\sum_{j=1}^{N} f_j Q_j P_j}$$

where:
f_i = Fraction of shares outstanding in the market float
w_i = Fraction of the portfolio that is allocated to security i or weight of security i
Q_i = Number of shares outstanding of security i
P_i = Share price of security i
N = Number of securities in the index

Fundamental Weighting

$$w_i^F = \frac{F_i}{\sum_{j=1}^{N} F_j}$$

where:
F_i = A given fundamental size measure of company i

Return Characteristics of Equity Securities

Total Return, $R_t = (P_t - P_{t-1} + D_t) / P_{t-1}$

where:
P_{t-1} = Purchase price at time t – 1
P_t = Selling price at time t
D_t = Dividends paid by the company during the period

Accounting Return on Equity

$$ROE_t = \frac{NI_t}{\text{Average } BVE_t} = \frac{NI_t}{(BVE_t + BVE_{t-1})/2}$$

Dividend Discount Model (DDM)

$$\text{Value} = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \ldots + \frac{D_\infty}{(1+k_e)^\infty}$$

$$\text{Value} = \sum_{t=1}^{n} \frac{D_t}{(1+k_e)^t}$$

One year holding period:

$$\text{Value} = \frac{\text{dividend to be received}}{(1+k_c)^1} + \frac{\text{year-end price}}{(1+k_c)^1}$$

Multiple-Year Holding Period DDM

$$V = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \ldots + \frac{P_n}{(1+k_e)^n}$$

where:
P_n = Price at the end of n years.

Infinite Period DDM (Gordon Growth Model)

$$PV_0 = \frac{D_0\,(1+g_c)^1}{(1+k_e)^1} + \frac{D_0\,(1+g_c)^2}{(1+k_e)^2} + \frac{D_0\,(1+g_c)^3}{(1+k_e)^3} + \ldots + \frac{D_0\,(1+g_c)^\infty}{(1+k_e)^\infty}$$

This equation simplifies to:

$$PV = \frac{D_0\,(1+g_c)^1}{(k_e - g_c)^1} = \frac{D_1}{k_e - g_c}$$

The long-term (constant) growth rate is usually calculated as:

$g_c = RR \times ROE$

Multi-Stage Dividend Discount Model

$$\text{Value} = \frac{D_1}{(1+k_e)^1} + \frac{D_2}{(1+k_e)^2} + \ldots + \frac{D_n}{(1+k_e)^n} + \frac{P_n}{(1+k_e)^n}$$

where:

$$P_n = \frac{D_{n+1}}{k_e - g_c}$$

D_n = Last dividend of the supernormal growth period
D_{n+1} = First dividend of the constant growth period

The Free-Cash-Flow-to-Equity (FCFE) Model

FCFE = CFO – FC Inv + Net borrowing

Analysts may calculate the intrinsic value of the company's stock by discounting their projections of future FCFE at the required rate of return on equity.

$$V_0 = \sum_{t=1}^{\infty} \frac{FCFE_t}{(1 + k_e)^t}$$

Value of a Preferred Stock

When preferred stock is non-callable, non-convertible, has no maturity date and pays dividends at a fixed rate, the value of the preferred stock can be calculated using the perpetuity formula:

$$V_0 = \frac{D_0}{r}$$

For a non-callable, non-convertible preferred stock with maturity at time, n, the value of the stock can be calculated using the following formula:

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1 + r)^t} + \frac{F}{(1 + r)^n}$$

where:
V_0 = value of preferred stock today (t = 0)
D_t = expected dividend in year t, assumed to be paid at the end of the year
r = required rate of return on the stock
F = par value of preferred stock

Price Multiples

$$\frac{P_0}{E_1} = \frac{D_1/E_1}{r - g}$$

$$\text{Price to cash flow ratio} = \frac{\text{Market price of share}}{\text{Cash flow per share}}$$

$$\text{Price to sales ratio} = \frac{\text{Market price per share}}{\text{Net sales per share}}$$

$$\text{Price to sales ratio} = \frac{\text{Market value of equity}}{\text{Total net sales}}$$

$$P/BV = \frac{\text{Current market price of share}}{\text{Book value per share}}$$

$$P/BV = \frac{\text{Market value of common shareholders' equity}}{\text{Book value of common shareholders' equity}}$$

where:
Book value of common shareholders' equity =
(Total assets - Total liabilities) - Preferred stock

Enterprise Value Multiples

EV/EBITDA

where:

EV = Enterprise value and is calculated as the market value of the company's common stock plus the market value of outstanding preferred stock if any, plus the market value of debt, less cash and short term investments (cash equivalents).

Fixed Income

Bond Coupon

Coupon = Coupon rate × Par value

Coupon Rate (Floating)

Coupon Rate = Reference rate + Quoted margin

Coupon Rate (Inverse Floaters)

Coupon rate = K – L× (Reference rate)

Callable Bond Price

Price of a callable bond = Value of option-free bond – Value of embedded call option

Putable Bond Price

Price of a putable bond = Value of option-free bond + Value of embedded put option

Dollar Duration

Dollar duration = Duration × Bond value

Inflation-Indexed Treasury Securities

TIPS coupon = Inflation adjusted par value × (Stated coupon rate/2)

Nominal spread

Nominal spread (Bond Y as the reference bond) = Yield on Bond X – Yield on Bond Y

Relative Yield spread

$$\text{Relative yield spread} = \frac{\text{Yield on Bond X} - \text{Yield on Bond Y}}{\text{Yield on Bond Y}}$$

Yield Ratio

$$\text{Yield ratio} = \frac{\text{Yield on Bond X}}{\text{Yield on Bond Y}}$$

After-Tax Yield

After-tax yield = Pretax yield × (1- marginal tax rate)

Taxable-Equalent Yield

$$\text{Taxable-equivalent yield} = \frac{\text{Tax-exempt yield}}{(1 - \text{marginal tax rate})}$$

Bond Value

$$\text{Bond Value} = \frac{\text{Maturity value}}{(1+i)^{\text{years till maturity} \times 2}}$$

where i equals the semiannual discount rate

Valuing a Bond Between Coupon Payments.

$$w = \frac{\text{Days between settlement date and next coupon payment date}}{\text{Days in coupon period}}$$

where:
w = Fractional period between the settlement date and the next coupon payment date.

$$\text{Present value}_t = \frac{\text{Expected cash flow}}{(1 + i)^{t-1+w}}$$

Current Yield

$$\text{Current yield} = \frac{\text{Annual cash coupon}}{\text{Bond price}}$$

Bond Price

$$\text{Bond price} = \frac{CPN_1}{\left(1 + \frac{YTM}{2}\right)} + \frac{CPN_2}{\left(1 + \frac{YTM}{2}\right)^2} + \frac{CPN_{2N} + Par}{\left(1 + \frac{YTM}{2}\right)^{2N}}$$

where:
Bond price = Full price including accrued interest.
CPN_t = The semiannual coupon payment received after t semiannual periods.
N = Number of years to maturity.
YTM = Yield to maturity.

Formula to Convert BEY into Annual-Pay YTM:

$$\text{Annual-pay yield} = \left[\left(1 + \frac{\text{Yield on bond equivalent basis}}{2}\right)^2 - 1\right]$$

Formula to Convert Monthly Cash Flow Yield into BEY

$$\text{BEY} = [(1 + \text{monthly CFY})^6 - 1] \times 2$$

Discount Basis Yeild

$$d = (1-p)\frac{360}{N}$$

Z-Spread

Z-spread = OAS + Option cost; and OAS = Z-spread - Option cost

Duration

$$\text{Duration} = \frac{V_- - V_+}{2(V_0)(\Delta y)}$$

where:
Δy = change in yield in decimal
V_0 = initial price
V_- = price if yields decline by Δy
V_+ = price if yields increase by Δy

Portfolio Duration

Portfolio duration = $w_1D_1 + w_2D_2 + \ldots + w_ND_N$

where:
N = Number of bonds in portfolio.
D_i = Duration of Bond i.
w_i = Market value of Bond i divided by the market value of portfolio.

Percentage Change in Bond Price

Percentage change in bond price = duration effect + convexity adjustment
$= \{[-\text{duration} \times (\Delta y)] + [\text{convexity} \times (\Delta y)^2]\} \times 100$

where:
Δy = Change in yields in decimals.

Convexity

$$C = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2}$$

Price Value of a Basis Point

Price value of a basis point = Duration × 0.0001 × bond value

DERIVATIVES

FRA Payoff

$$\frac{\text{Floating rate at expiration} - \text{FRA rate} \times (\text{days in floating rate}/\ 360)}{1 + [\text{Floating rate at expiration} \times (\text{days in floating rate}/\ 360)}$$

Numerator: Interest savings on the hypothetical loan. This number is positive when the floating rate is greater than the forward rate. When this is the case, the long benefits and expects to receive a payment from the short. The numerator is negative when the floating rate is lower than the forward rate. When this is the case, the short benefits and expects to receive a payment from the long.

Denominator: The discount factor for calculating the present value of the interest savings.

Call Option Payoffs

Option Position	Description	Payoffs	
		$S_T > X$	$S_T < X$
		Option holder exercises the option.	Option holder does not exercise the option.
Call option holder	Choice to buy the underlying asset for X	$S_T - X$	0
Call option writer	Obligation to sell the underlying asset for X if the option holder chooses to exercise the option	$-(S_T - X)$	0

Intrinsic Value of a Call Option

Intrinsic value of call = Max $[0, (S_t - X)]$

Put Option Payoffs

Option Position	Description	Payoffs	
		$S_T < X$	$S_T > X$
		Option holder exercises the option	Option holder does not exercise the option
Put option holder	Choice to sell the underlying asset for X	$X - S_T$	0
Put option writer	Obligation to buy the underlying asset for X if the option holder chooses to exercise the option	$-(X - S_T)$	0

Moneyness and Intrinsic Value of a Put Option

Moneyness	Current Market Price (S_t) versus Exercise Price (X)	Intrinsic Value Max $[0, (X - S_t)]$
In-the-money	S_t is less than X	$X - S_t$
At-the-money	S_t equals X	0
Out-of-the-money	S_t is greater than X	0

Option Premium

Option premium = Intrinsic value + Time value

Put-Call Parity

$$C_0 + \frac{X}{(1 + R_F)^T} = P_0 + S_0$$

Synthetic Derivative Securities

Strategy	Consisting of	Value	Equals	Strategy	Consisting of	Value
fiduciary call	long call + long bond	$C_0 + \frac{X}{(1 + R_F)^T}$	=	Protective put	long put + long underlying asset	$P_0 + S_0$
long call	long call	C_0	=	Synthetic call	long put + long underlying asset + short bond	$P_0 + S_0 - X/(1+R_F)^T$
long put	long put	P_0	=	Synthetic put	long call + short underlying asset + long bond	$C_0 - S_0 + X/(1+R_F)^T$
long underlying asset	long underlying asset	S_0	=	Synthetic underlying asset	long call + long bond + short put	$C_0 + X/(1+R_F)^T - P_0$
long bond	long bond	$\frac{X}{(1 + R_F)^T}$	=	Synthetic bond	long put + long underlying asset + short call	$P_0 + S_0 - C_0$

Option Value Limits

Option	Minimum Value	Maximum Value
European call	$EC_t \geq 0$	$EC_t \leq S_t$
American call	$AC_t \geq 0$	$AC_t \leq S_t$
European put	$EP_t \geq 0$	$EP_t \leq X/(1 + RFR)^T$
American put	$AP_t \geq 0$	$AP_t \leq X$

Option Value Bounds

Option	Minimum Value	Maximum Value
European Call	$\text{Max}\left[0, S_t - \frac{X}{(1+RFR)^T}\right]$	S_t
American Call	$\text{Max}\left[0, S_t - \frac{X}{(1+RFR)^T}\right]$	S_t
European Put	$\text{Max}\left[0, \frac{X}{(1+RFR)^T} - S_t\right]$	$\frac{X}{(1+RFR)^T}$
American Put	$\text{Max}\left[0, X - S_t\right]$	X

Interest Rate Call Holder's Payoff

$$= \text{Max}\,(0, \text{Underlying rate at expiration} - \text{Exercise rate})\left(\frac{\text{Days in underlying Rate}}{360}\right) \times NP$$

where: NP = Notional principal

Interest Rate Put Holder's Payoff

$$= \text{Max}\,(0, \text{Exercise rate} - \text{Underlying rate at expiration})\left(\frac{\text{Days in underlying rate}}{360}\right) \times NP$$

where:
NP = Notional principal

Net Payment for a Fixed-Rate-Payer

Net fixed-rate payment$_t$ = (Swap fixed rate - LIBOR$_{t-1}$)×(No. of days/360)×(NP)
where:
NP equals the notional principal.

Summary of Options Strategies

	Call	Put
Holder	$C_T = \max(0,S_T - X)$ Value at expiration = C_T Profit: $\Pi = C_T - C_0$ Maximum profit = ∞ Maximum loss = C_0 Breakeven: $S_T^* = X + C_0$	$P_T = \max(0,X - S_T)$ Value at expiration = P_T Profit: $\Pi = P_T - P_0$ Maximum profit = $X - P_0$ Maximum loss = P_0 Breakeven: $S_T^* = X - P_0$
Writer	$C_T = \max(0,S_T-X)$ Value at expiration = $-C_T$ Profit: $\Pi = -C_T - C_0$ Maximum profit = C_0 Maximum loss = ∞ Breakeven: $S_T^* = X + C_0$	$P_T = \max(0,X - S_T)$ Value at expiration = $-P_T$ Profit: $\Pi = -P_T - P_0$ Maximum profit = P_0 Maximum loss = $X - P_0$ Breakeven: $S_T^* = X - P_0$

Where:

C_0, C_T = price of the call option at time 0 and time T
P_0, P_T = price of the put option at time 0 and time T
X = exercise price
S_0, S_T = price of the underlying at time 0 and time T
V_0, V_T = value of the position at time 0 and time T
Π = profit from the transaction: $V_T - V_0$
r = risk-free rate

Covered Call

Value at expiration: $V_T = S_T - \max(0,S_T - X)$
Profit: $\Pi = V_T - S_0 + C_0$
Maximum profit = $X - S_0 + C_0$
Maximum loss = $S_0 - C_0$
Breakeven: $S_T^* = S_0 - C_0$

Protective Put

Value at expiration: $V_T = S_T + \max(0,X - S_T)$
Profit: $\Pi = V_T - S_0 - P_0$
Maximum profit = ∞
Maximum loss = $S_0 + P_0 - X$
Breakeven: $S_T^* = S_0 + P_0$